a/ * d'après 1) $e^{\frac{1}{n}} > 1 + \frac{1}{n}$

$\Rightarrow \left(e^{\frac{1}{n}}\right)^n > (1 + \frac{1}{n})^n \Rightarrow \boxed{e > U_n}$ (1)

* $\frac{1}{n+1} \in]0, 1[$ d'après (ii)

$e^{\frac{1}{n+1}} < \frac{1}{1 - \frac{1}{n+1}} \Rightarrow e^{\frac{1}{n+1}} < 1 + \frac{1}{n}$

$\Rightarrow (e^{\frac{1}{n+1}})^{n+1} < (1 + \frac{1}{n})^{n+1}$

$\Rightarrow \boxed{e < V_n}$ (2)

(1) + (2) $\Rightarrow$ $U_n < e < V_n$, $\forall n \in$ IN*

b/ $V_n - U_n = (1 + \frac{1}{n})^{n+1} - (1 + \frac{1}{n})^n$

$= [(1 + \frac{1}{n}) - 1](1 + \frac{1}{n})^n$

$= [(1 + \frac{1}{n}) - 1]U_n$

$V_n - U_n = \frac{1}{n}U_n$, $n \in$ IN*

$U_n \leq e \Rightarrow U_n \leq 3 \Rightarrow \frac{1}{n}U_n \leq \frac{3}{n}$

D'où $V_n - U_n \leq \frac{3}{n}$, pour $n \in$ IN*

c/ $V_n - U_n \leq \frac{3}{n}$, $n \in$ IN*

$\frac{3}{n} \leq 10^{-6} \Leftrightarrow \frac{n}{3} \geq 10^6 \Leftrightarrow n \geq 3.10^6$

D'où $U_{3\,000\,000} < e < V_{3\,000\,000}$ (calculatrice)

$\Rightarrow 2{,}718254 < e < 2{,}718255$

EX17 :

1) $A = \int_0^1 \frac{e^x}{1+e^x} dx =$

$[Ln(1+e^x)]_0^1$ $\boxed{A = Ln(\frac{1+e}{2})}$

$B = \int_0^1 \frac{e^x}{(1+e^x)^2} dx = \left[\frac{-1}{1+e^x}\right]_0^1 = \frac{1}{2} - \frac{1}{e+1}$ $\boxed{B = \frac{e-1}{2(e+1)}}$

2) $t > 0$

$\frac{1}{(1+t)^2} = \frac{(1+2t+t^2)-(t^2+2t)}{(1+t)^2}$

$= 1 - \frac{t^2+2t}{(1+t)^2} = 1 - \frac{(t^2+t)+t}{(1+t)^2}$

$= 1 - (\frac{t}{1+t} + \frac{t}{(1+t)^2})$

$= 1 - \frac{t}{1+t} - \frac{t}{(1+t)^2}$

$(a = 1, b = -1 ; c = -1)$

$I = \int_0^1 (1 - \frac{e^x}{1+e^x} - \frac{e^x}{(1+e^x)^2}) dx$

$I = \int_0^1 dx - \int_0^1 \frac{e^x}{1+e^x} dx - \int_0^1 \frac{e^x}{(1+e^x)^2} dx$

$I = [x]_0^1 - (A + B)$

$I = 1 - (Ln(\frac{1+e}{2}) + \frac{1}{2} - \frac{1}{e+1})$

$\boxed{I = 1 - (Ln(\frac{1+e}{2}) + \frac{e-1}{2(e+1)})}$

3) $J = \int_0^1 \frac{xe^x}{(1+e^x)^3} dx$

Qarray.tn -Aqra Blech- Qarray.tn